

All of the UK curricula define multiple categories of mathematical proficiency that require pupils to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths' (PTM)* categories are based on the *Curriculum Aims* in the KS1, KS2 and KS3 National Curriculum for England (2013), and are also comparable with the GCSE Assessment Objectives: they adopt some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Pupils can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Pupils can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (for example, calculate with or compare decimal numbers, identify odd numbers, prime numbers and multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Pupils can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Pupils can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*, especially at younger ages where reading and English comprehension restrict the sorts of questions that can be asked. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

This table shows how the questions in *PTM11* map onto these process categories.

Process categories	Mental Maths	Applying and Understanding Maths
FF: Fluency in facts and procedures	1, 2, 4, 5, 7, 10	1a, 6, 17
FC: Fluency in conceptual understanding	3, 6, 8, 12, 13, 14, 16	4, 5a, 13b, 18a, 20, 21a, 21b, 23, 24, 25, 28b, 29a
MR: Mathematical reasoning	9, 11, 15, 17, 18, 19, 20	1b, 2, 3, 5b, 8, 9, 10, 11, 12, 13a, 15, 16, 19, 26, 28a, 29b, 31
PS: Problem solving		7, 14, 18b, 22, 27, 30, 32

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories shown above are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest <i>PTM</i> process categories			
Key Stage 2 Skills	FF	FC	MR	PS
1. Solve mathematical problems				•
2. Communicate mathematically		•	•	
3. Reason mathematically		•	•	
Foundation Phase Range	•			

Northern Ireland	Closest <i>PTM</i> process categories			
Key Stage 2 Processes in Mathematics	FF	FC	MR	PS
Making and monitoring decisions				•
Communicating mathematically		•	•	
Mathematical reasoning		•	•	•
Individual mathematical topics	•			

Scotland *	Closest PTM process categories			
Experiences and outcomes	FF	FC	MR	PS
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts				
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts			•	•

* Education Scotland 'Curriculum for Excellence: Numeracy and Mathematics' 14 May 2009.
 Accessed: 31 July 2014. www.curriculumforexcellencescotland.gov.uk

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the pupils in the relevant age group. The activities may therefore be used to obtain diagnostic information about each pupil's strengths and weaknesses, and may also be used to provide a basis from which pupils' mathematical understanding may be further developed.

This section discusses some of the ways in which pupils may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to pupils and discussing the activities in which they did well, in addition to those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the Pupil Booklet and the Teacher's script in the At a Glance Guide when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the overall performance of your pupils, and a basis for progress monitoring. This section is intended to help you identify the specific difficulties that pupils have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test pupils' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills and are relying too heavily on written methods for even simple sums. They may lack the confidence to recall mathematical facts under time pressure.

Regular quick-fire quizzes may help pupils gain fluency and confidence, and there are many software products that allow students to practise skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help pupils gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper and digital test

Question 1: Number sequence

Pupils are given the first three terms in a sequence in which each term is five more than the previous term. They are asked to find the next two numbers in the sequence (part a) and to circle the three numbers in the list which would fit into the same sequence (part b).

Classroom activities in which pupils work with such sequences (possibly showing them on number lines or in number grids), not just continuing them according to a rule, but thinking about which numbers could occur in the sequences and finding explanations for their ideas, are helpful. This provides excellent opportunities for mathematical reasoning and explanations, which are important aims of the new curriculum.

Question 2: Divide by five

Pupils are shown the calculation $88 \div 5 = 17.6$ and asked to answer different real-life calculations based on this. Firstly, they are asked how many £5 tickets can be bought for £88 and are required to round their answer down (part a); next they are asked how many 5-seater taxis are needed for 88 people, and so must round their answer up (part b). Students are then asked how much each person would get if £88 were divided between five people (part c).

These are all realistic ways in which the answer to such a division calculation may need to be interpreted in the real world. Pupils can benefit by not just practising divisions on paper and using a calculator, but also by discussing what the result of a division calculation means in different circumstances. Asking pupils whether they can find other situations in which the given calculation could apply can be a very fruitful activity. Pupils can also make up similar problems for themselves.

Question 3: Postage

In this task, pupils are provided with a table which shows the various costs of posting parcels of different weights. Pupils have to find how much it would cost to post one parcel weighing 190 grams and two parcels each weighing 520 grams. The pupils need to select the correct number cards, from the list provided, to construct the correct calculation and then work out the answer.

Finding the correct cost of posting three parcels, using the given weights of the parcels and a table providing prices for different weights proved quite challenging, with less than half of pupils finding a correct solution to the problem. In class, pupils can work with such tables, including those that they have made for themselves, such as tables giving driving distances, bus timetables, ingredients in

foods, and numbers of boys and girls in different years at the school. Collecting their own data can provide opportunities for pupils to draw graphs and pictograms and then use these to solve problems.

Question 4 and 5: Sweets

The task concerns simple fractions in the context of sets of differently coloured sweets. In question 4, pupils are provided with a diagram showing twelve sweets of different colours: red, blue, yellow and green. They are asked what fraction of the sweets are red (part a) and not red (part b). In question 5, pupils are told that Sam, Raj and Jade also have twelve sweets each. They are asked to calculate one third of this amount (part a) and then select which fraction represents the largest number of blue sweets (part b).

To develop their skills with such questions further, there are many situations involving people, counters, books, money, and so on, that pupils can work with, and pupils can ask and answer questions involving decimals and percentages as well as fractions.

Question 6 and 7: Triangle angles

Pupils are provided with a diagram showing four triangles in which some of the angles are given. They are asked to identify the equilateral triangle (question 6) and then to circle the two triangles that have the same angles as each other (question 7).

Such calculations can be set in the context of solving simple algebraic equations. Pupils enjoy and benefit from drawing and measuring sides and angles of triangles and other shapes, perhaps outside in the playground, in addition to working on classroom exercises. Identifying different shapes in tessellations and other designs, or in solid shapes, and calculating their angles, finding their lines of symmetry and describing their other properties can be valuable.

Question 8 - 11: School

Pupils are provided with four graphs which show information about pupils at the Hill School but have no labels. They are given clues about the trends in the different categories and are asked to identify the graph which shows the total number of pupils of each age (question 8); the graph that shows the number of pupils who have school dinners (question 9); the graph that shows the number of pupils who walk to school (question 10) and the graph which shows the average height of pupils (question 11).

Questions such as this provide an excellent opportunity for pupils to explain their reasons for allocating each graph to a particular description; they may also suggest other data that may fit the shapes of the graphs, drawing on their own experiences.

Question 12: Sale price

Pupils are shown four coats which all cost £33 originally and each have a different sale label. From the four options provided, pupils must match each new sale price with the correct coat, according to the amount that their price was reduced by.

Pupils should be able to solve calculations involving percentages, and money problems involving fractions. Pupils can collect their own examples from shops, newspapers and magazines, and the internet, to work out reductions and to calculate the percentage or fraction reductions if they are given the original and the reduced prices. Some pupils may need help in interpreting the words and numbers; one common error is to confuse 'one third of' with 'one third off', for example. They may also need practice in converting between fractions and percentages.

Question 13: Pyramid

Pupils are provided with four diagrams showing nets of pyramids. The task is to identify the net which will not fold up to make a square based pyramid (part a) and state how many vertices a pyramid on a square base has (part b).

Pupils should be able to recognise, describe and build simple 3D shapes, including making nets. In class, pupils need to get plenty of experience of making 3D shapes from nets and opening up 3D shapes to see their nets. Pupils need to identify the number of faces, edges and vertices of real solids, from 2D representations of solids, as well as mentally without any picture or object. Such practice will help pupils to visualise different aspects of the shapes and will support work on volume and surface area. Pupils may find it interesting to explore which 3D shapes are used in packaging and why.

Question 14: Boys and girls

This question asks how many pupils there are altogether in a class, given that there are sixteen girls and twice as many girls as boys. Pupils are asked to use the number cards provided to write the correct calculation for working out the total number of pupils in the class.

Pupils should be taught to solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division. There are many ways that such problems can be tackled and pupils need to be given an opportunity to explore their ideas in small group discussion, perhaps using counters of different colours to represent girls and boys. Small variations on the same problem can help pupils to see generalities. Ratio and proportion are important underlying ideas in much of the National Curriculum and pupils need to be helped to see how the same mathematics can enable them to work on apparently different problems.

Question 15: Notebooks

Pupils are provided with a diagram showing seven coins. They are asked how many 45p notebooks Amy can buy (part a); and how much money will she have left (part b). Classroom buying and selling activities with real or artificial coins can help to develop speed and accuracy with money. Pupils may find a number line helpful for totalling the money and then for finding how much is left. Teachers may include such activities as part of a larger activity on money management in general.

Question 16: Driving

A table showing driving distances between six towns is provided. Pupils are asked to find how many miles Joe drives when he travels from Edinburgh to Birmingham, then on to London (part a) and then to find how much further this is than the direct journey from Edinburgh to London (part b).

As mentioned earlier, there are many examples of tables in everyday life and it is important that pupils learn how to interpret them and use them to solve problems. Working with such tables could form part of an activity about planning a real trip, where pupils can calculate distances, times and costs.

Question 17 and 18: Necklaces

Diagrams showing how two different types of beads are used to make necklaces are provided. Pupils are asked to complete a table to show the number of each type of bead that is needed to make four triangles (question 17); they then need to complete two rules for every time a new triangle is made (question 18a) and finally they need to find how many triangles there are in a necklace with twenty-one long beads (question 18b).

Pupils should be able to generate and describe linear number sequences. Pupils enjoy working on geometrical and numerical patterns such as this, and their work can be extended into generalisation, explanation and the use of algebraic notation.

Question 19: Wheel

A diagram showing a wheel with eight evenly spaced spokes is shown. Pupils are asked to calculate the angle between the spokes (part a) and to find how many spokes there are on a wheel that has an angle of 40° between two adjacent spokes (part b).

In the classroom, pupils can work in various ways with circles and radii, folding paper to make symmetrical patterns, drawing regular polygons with their vertices on circles, exploring turning angles, and so on.

Question 20 - 22: Factor Bingo

In this task, two bingo cards are shown. The rules state that when a number is called out all the factors of that number can be crossed out. All of the factors of eight are crossed out except one and pupils are asked to cross out the missing factor (question 20). Pupils are then told that the next number is thirty-six and are asked to cross off the factors for this number on both bingo cards (question 21a); they then do the same for the next number, twenty-eight (question 21b). Pupils are then informed that the computer has shown another number and somebody has shouted 'I win': using deduction, pupils must identify who the winner is (question 22a). Finally, the pupils are told the number is less than fifty and are asked to work out what it is (question 22b).

Pupils need to be fluent with finding the factors and multiples of numbers; playing games such as this can provide useful practice in an enjoyable setting. Shading multiples of numbers on 10 by 10 number grids, and then grids with different widths, is useful for exploring and explaining patterns (for example, the multiples of which numbers on a 12 by 12 grid have the same pattern as the multiples of two and five on the 10-by-10 grid? Why?). Factor trees can also help pupils to gain fluency. Can pupils explain why only square numbers have an odd number of factors?

Question 23: Brackets

In this question, pupils are asked to put brackets into two calculations to make them correct. Pupils should be able to use their knowledge of the order of operations to carry out calculations involving the four operations.

Most calculators are programmed to use the conventional order and will give the correct answers. If some pupils have calculators that give the correct answers and others do not, this can stimulate some interesting discussions which make clear that there is a need for conventions. At this stage, pupils need to understand clearly that brackets take precedence, and that multiplication and division are done before addition and subtraction. Pupils can invent their own calculations, as complicated as they like, and explore how many different answers they can make by putting brackets in different places. Such activities can encourage creativity as well as systematic working.

Question 24: Number line

In this question, pupils are asked to identify where four simple fractions should be placed on a number line. Each of the relevant points has a blank label and candidates need to allocate fractions to labels.

Fraction misconceptions abound and pupils need to have classroom experiences that allow such misconceptions to be revealed, explored and discussed so that they can be overcome. A fraction wall can help pupils to compare all these fractions and arrange them in order of size.

Question 25 - 27: Zainab's biscuits

In this task, pupils are provided with a recipe for making enough biscuits for two people. Pupils are asked what fraction of all the ingredients sugar is (question 25); what the ratio of sugar is to flour (question 26) and how much flour is needed to make enough biscuits for four people (question 27).

Recipes provide a rich context for work on units, reading scales, fractions, ratios and percentages, so it is worthwhile using them in the classroom so that pupils can come to understand that the proportions of ingredients (expressed as fractions, ratios or percentages) in a recipe remain the same when the recipe is scaled up or down for different numbers of people. Other useful contexts are the population of the school (boys/girls, ages, infant/junior, pupils/teachers); how I spend my day / money, and so on. Please also see the comments about questions 31 and 32, Charity.

Question 28: Square co-ordinates

A diagram showing three corners of a square on a square grid is shown. Pupils are asked to mark the fourth corner (part a); and give its coordinates (part b).

There are further tasks like this that can be done in the classroom: given two vertices of a square, in how many ways can the square be completed? Are all the squares the same size? What areas of squares can you make? What about non-square rectangles and what about rhombuses?

Question 29 and 30: Pocket money

In this question, pupils are given a word formula for calculating the pocket money of a child of a particular age. Pupils are asked to use the rule to work out the pocket money of a seven-year-old (question 29a); pupils then need to use the formula backwards to find the age corresponding to a given amount of pocket money (question 29b); finally, pupils need to select the algebraic formula which expresses this rule (question 30).

Word formulas can be explored in the classroom, with pupils finding and examining patterns in the correct answers. They could make graphs and use the graph to answer questions. They could ask 'What if?' questions and see how changing the word formula affects the graph and the algebraic formula.

Question 31 and 32: Charity

A pie chart is provided showing how a charity spends its money. Pupils are asked to label the sectors of the pie chart using the information given about the charity's spending (question 31) and are then asked to calculate how much of a £160 donation will be spent on office work (question 32).

Again, this question is about ratio, which earlier questions have shown is a challenging new area for many pupils. Pie charts are visual representations of proportions and hence provide a good context for working on them, as well as the examples given in the notes on questions 25-27, Zainab's biscuits.

Feedback to parents and carers

A report on the individual pupil is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the pupil at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual pupil in order to understand and maximise their potential.
- Explain that testing with *PTM11* is part of the school's regular assessment regime and that all pupils in the year group(s) have been tested.
- As part of the test, pupils were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual pupil (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM11*.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our pupils to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 11, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.