Mathematics process categories Form A

PT MATHS 8

All of the UK curricula define multiple categories of mathematical proficiency that require pupils to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths' (PTM)* categories are based on the *Curriculum Aims* in the KS1, KS2 and KS3 National Curriculum for England (2013), and are also comparable with the GCSE Assessment Objectives: they adopt some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Pupils can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Pupils can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (for example, calculate with or compare decimal numbers, identify odd numbers, prime numbers and multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Pupils can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Pupils can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*, especially at younger ages where reading and English comprehension restrict the sorts of questions that can be asked. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

These tables show how the questions in *PTM*8 map onto these process categories.

Paper test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	1, 2, 8, 10, 11, 12	1, 3, 6, 10, 12
FC: Fluency in conceptual understanding	5, 6, 13, 14	2a, 4, 5a, 8a, 9, 10, 11a, 14a
MR: Mathematical reasoning	3, 4, 7, 9, 15	2b, 5b, 8b, 11b, 14b, 14c, 7a, 7b
PS: Problem solving		7c, 11c, 11d, 13

Digital test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	1, 2, 8, 10, 11, 12	1a, 1b, 3, 4, 5, 8, 19
FC: Fluency in conceptual understanding	5, 6, 13, 14	2a-c, 6a-c, 6d, 7a, 12a, 13, 14, 18, 19, 21
MR: Mathematical reasoning	3, 4, 7, 9, 15	2d, 7b, 9, 10, 12b, 15, 19, 22a, 22b
PS: Problem solving		11, 16, 17, 20

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories shown above are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 2 Skills	**FF**	**FC**	**MR**	**PS**
1. Solve mathematical problems				•
2. Communicate mathematically		•	•	
3. Reason mathematically		•	•	
Foundation Phase Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 1 Processes in Mathematics	**FF**	**FC**	**MR**	**PS**
Making and monitoring decisions				•
Communicating mathematically		•	•	
Mathematical reasoning		•	•	•
Individual mathematical topics	•			

Scotland *	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

* Education Scotland 'Curriculum for Excellence: Numeracy and Mathematics' 14 May 2009. Accessed: 31 July 2014. www.curriculumforexcellencescotland.gov.uk

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the pupils in the relevant age group. The activities may therefore be used to obtain diagnostic information about each pupil's strengths and weaknesses, and may also be used to provide a basis from which pupils' mathematical understanding may be further developed.

This section discusses some of the ways in which pupils may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to pupils and discussing the activities in which they did well, in addition to those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the Pupil Booklet and the Teacher's script in the At a Glance Guide when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the overall performance of your pupils, and a basis for progress monitoring. This section is intended to help you identify the specific difficulties that pupils have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test pupils' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills and are relying too heavily on written methods for even simple sums. They may lack the confidence to recall mathematical facts under time pressure.

Regular quick-fire quizzes may help pupils gain fluency and confidence, and there are many software products that allow students to practise skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help pupils gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper Test

Question 1: Total 100

In this question, pupils are asked to pair up numbers that add up to 100. The idea of complementarity is useful and pupils can be given opportunities to practise complements in a variety of contexts such as fractions and giving change using both £ and p in contexts, as well as in whole number examples such as this question.

Question 2: Where does Pete live?

The problem is to find Pete's house number given four clues. The number is bigger than 24 but less than 46, an odd number (part a) and divisible by 5 (part b). Pupils enjoy solving puzzles such as this in the classroom, to provide practice in reading and understanding the conditions that have to be satisfied. The use of a 10 by 10 number grid can assist pupils in identifying which numbers fit the conditions and provides visual illustrations of sets of numbers such as odd numbers and multiples of five.

Question 3: Bricks

This activity asks pupils to circle two piles of bricks with a total of 10 (part a) tick three piles with a total of 9 (part b); then to make three equal piles of bricks (part c).

Pupils can benefit from an opportunity to work on tasks of this kind with simple classroom resources such as counters or multilink, setting problems for each other. This can also help to develop both numeracy and literacy skills.

Question 4: Shading fractions

In this question pupils are asked to shade one half of a circle, one third of a circle, two sixths of a circle (part a); and finally to state which two of the fractions are the same size (part b).

Fractions are one of the most challenging aspects of the mathematics curriculum for many pupils.

Pupils could spend time working with practical apparatus and with fractions represented in different ways and in different contexts such as time (halves and quarters of an hour, and more complex fractions using minutes) and equal shares (the diagrams on the test page can be interpreted as pizzas, for example) to help them gain understanding and fluency. Pupils need time to discuss answers and sort out errors and misconceptions. Pupils should count up in halves and quarters with the support of a number line, to help them to see fractions as numbers.

Question 5: Pocket money

This task requires pupils to find the total of 10 coins of different values in £ and p (part a); then to find the change from £6.87 after spending £3.45 (part b). Pupils should be able to add and subtract amounts of money to give change, using both £ and p.

In a practical situation where they can move the coins around, pupils are likely to be more successful, but they need experience of dealing with 'static' money such as presented here, perhaps ticking off each coin as they count it, and/or listing the amounts to help to find the total. When finding the amount left over, applying counting-on to practical situations and using an empty number line (and exploring the similarities between these two methods) may be helpful. Pupils need to gain as much real experience of working with money as possible.

Question 6: Shapes

This question asks pupils to draw lines connecting shapes to their names (part a); then write the names of two shapes (part b). Pupils benefit from lots of experience of working with and talking about regular and irregular polygons as part of their mathematics and other lessons, so that they learn that, for example, not all hexagons are the same shape, and that (apart from triangles) polygons can have all sides equal but not be regular. Teachers need to be careful always to use the correct vocabulary themselves and to help pupils to use it correctly. Polygons can be used in shape-sorting exercises, to make designs, and as shapes to split up into different fractions, and so on.

Question 7: Digit cards

In this task, pupils need to arrange four cards to make calculations such that the sum of two two-digit numbers is as large as possible (part a); the sum is as small as possible (part b); then make a subtraction calculation with the largest possible answer (part c).

Pupils should be taught to solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Pupils with a good understanding of place value are able to decide how to place the four cards to make two two-digit numbers with the largest and smallest sums and differences. Others will perhaps feel the need to try a number of possibilities, which is not practicable under test conditions, but can be explored in the classroom, so that pupils can come to understand the importance of the position of each digit in the calculations. In the classroom they can also explore if their answer is unique or if there are a number of possibilities and, if so, why.

Ideas of place value can be strengthened by games, such as placing random digits as they are called out by the teacher so that the final number is as big (or as small) as possible. Pupils holding single-digit cards can arrange themselves in a line to make the biggest/smallest possible number and explain why they are correct. Teachers may also wish to use structured apparatus to provide rich experiences of place value. Clearly it is very important for pupils to gain a sound understanding of how the number system works.

Question 8: L-shapes

In this task pupils are provided with a diagram showing three L-shapes on a square grid. Pupils are asked to count the number of squares in each L-shape (part a); then to state how many squares are needed to make the next L-shape (part b). In class, teachers could give pupils an opportunity to work systematically and to observe, describe and explain mathematical patterns linking arithmetic and geometry.

Question 9: Pencils

In this task pupils are provided with a list of information showing children's names and the number of pencils they have; they then have to complete a table to show this information.

Although there is no 'total' row for the table, pupils should be encouraged to always check their work by making sure that the number of children represented in the table is the same as that given in the question, to eliminate possible errors. Pupils need to be provided with a range of classroom opportunities to collect and organise their own data.

Question 10: Spider

In this activity, pupils are provided with eight calculations and asked to identify those that don't have the answer 24.

This number spider offers a variety of calculations resulting in the number 24, some of which are incorrect; pupils are asked to identify the incorrect calculations.

Making up number spiders such as this is an enjoyable and creative activity for pupils. It can be tackled at many different levels to gain confidence and fluency in calculations. At different ages, pupils can be asked to include fractions, decimals, percentages, indices, and so on, or more elementary calculations according to what they need to work on. It can also provide a focus for order of operations and for using calculators when these are felt to be appropriate.

Question 11: Measures

This is a four-part question about measures. Pupils are asked to decide which pencil is about 14cm long (part a); how tall is Victoria's teacher (part b); how much

does one blueberry weigh (part c); how many grapes weigh the same as one orange (part d).

For parts a and b, pupils need plenty of practice in realistic situations, to eliminate errors such as starting at 1 on the ruler instead of zero, and to build up a knowledge of the approximate size of common objects, such as the height of a door being two metres. Units are often not well understood or recognised; practical work can help pupils to gain familiarity with centimetres and metres. Practical activities using a variety of different weighing scales and everyday objects are necessary to improve pupils' understanding of weight/mass.

Question 12: Levi's homework

In this question, pupils need to solve four missing number problems: an addition problem (part a); a subtraction problem (part b); a multiplication problem (part c); and a division problem (part d).

Pupils should be taught to solve addition, subtraction, multiplication and division missing number problems.

Pupils need practice in solving questions like this with the missing number in all possible positions, and they need to be encouraged to check that their answers work. They will also benefit from the opportunity to make up such problems for their fellow pupils to solve.

Question 13: Sharing

In this problem, pupils are asked to calculate the total number of sweets if each of nine children has four sweets and there is one left over.

If pupils are to use a more sophisticated method using multiplication, they need to know their tables and how and when to apply these number facts. Exploring different ways of solving such problems in class, with lots of discussion between pupils as they explain and evaluate the methods they use, is extremely valuable.

Question 14: Authors

This question provides a pictogram showing pupils' favourite authors. The key shows that one icon represents 10 pupils. Pupils are asked to find the most popular author (part a); how many chose Michael Rosen as their favourite (part b); how many took part in the survey (part c).

In the classroom, pupils enjoy, and benefit from, collecting their own data and representing it as pictograms, choosing appropriate keys for the information collected. They can also look through newspapers and magazines and on the internet for examples of pictograms to be discussed and interpreted in class.

Digital Test

Question 1: Total 100

In this question, pupils are asked to pair up numbers that add up to 100. The idea of complementarity is useful and pupils can be given opportunities to practise complements in a variety of contexts such as fractions and giving change using both £ and p in contexts, as well as in whole number examples such as this question.

Question 2: Where does Pete live?

The problem is to find Pete's house number given four clues. The number is bigger than 24 (part a) but less than 46 (part b), an odd number (part c) and divisible by 5 (part d). Pupils enjoy solving puzzles such as this in the classroom, to provide practice in reading and understanding the conditions that have to be satisfied. The use of a 10 by 10 number grid can assist pupils in identifying which numbers fit the conditions and provides visual illustrations of sets of numbers such as odd numbers and multiples of five.

Questions 3, 4 and 5: Bricks

This activity asks pupils to click on two piles of bricks with a total of 10 (question 3) click on three piles with a total of 9 (question 4); then to answer how many bricks there would be in each pile if there were three equal piles of bricks (question 5). Pupils can benefit from an opportunity to work on tasks of this kind with simple classroom resources such as counters or multilink, setting problems for each other. This can also help to develop both numeracy and literacy skills.

Question 6: Shading fractions

In this question pupils are asked to shade one half of a circle (part a), one third of a circle (part b), two sixths of a circle (part c); and finally to state which two of the fractions are the same size (part d).

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Question 8: Shapes

This question asks pupils to move the shapes into the correct, named boxes. Pupils benefit from lots of experience of working with and talking about regular and irregular polygons as part of their mathematics and other lessons, so that they learn that, for example, not all hexagons are the same shape, and that (apart from triangles) polygons can have all sides equal but not be regular. Teachers need to be careful always to use the correct vocabulary themselves and to help pupils to use it correctly. Polygons can be used in shape-sorting exercises, to make designs, and as shapes to split up into different fractions, and so on.

Questions 9, 10 and 11: Digit cards

In this task, pupils need to arrange four cards to make calculations such that the sum of two two-digit numbers is as large as possible (question 9); the sum is as small as possible (question 10); then make a subtraction calculation with the largest possible answer (question 11).

Pupils should be taught to solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Pupils with a good understanding of place value are able to decide how to place the four cards to make two two-digit numbers with the largest and smallest sums and differences. Others will perhaps feel the need to try a number of possibilities, which is not practicable under test conditions, but can be explored in the classroom, so that pupils can come to understand the importance of the position of each digit in the calculations. In the classroom they can also explore if their answer is unique or if there are a number of possibilities and, if so, why.

Ideas of place value can be strengthened by games, such as placing random digits as they are called out by the teacher so that the final number is as big (or as small) as possible. Pupils holding single-digit cards can arrange themselves in a line to make the biggest/smallest possible number and explain why they are correct. Teachers may also wish to use structured apparatus to provide rich experiences of place value. Clearly it is very important for pupils to gain a sound understanding of how the number system works.

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Question 13: Pencils

In this task pupils are provided with a list of information showing children's names and the number of pencils they have; they then have to complete a table to show this information.

Although there is no 'total' row for the table, pupils should be encouraged to always check their work by making sure that the number of children represented in the table is the same as that given in the question, to eliminate possible errors. Pupils need to be provided with a range of classroom opportunities to collect and organise their own data.

Question 14: Spider

In this activity, pupils are provided with eight calculations and asked to identify those that don't have the answer 24.

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Making up number spiders such as this is an enjoyable and creative activity for pupils. It can be tackled at many different levels to gain confidence and fluency in calculations. At different ages, pupils can be asked to include fractions, decimals, percentages, indices, and so on, or more elementary calculations according to what they need to work on. It can also provide a focus for order of operations and for using calculators when these are felt to be appropriate.

Questions 15, 16, 17 and 18: Measures

This is a four-part question about measures. Pupils are asked to decide how tall is Victoria's teacher (question 15); how much does one blueberry weigh (question 16); how many grapes weigh the same as one orange (question 17) and to identify the length of three pencils (question 18).

For questions 15 and 18, pupils need plenty of practice in realistic situations, to eliminate errors such as starting at 1 on the ruler instead of zero, and to build up a knowledge of the approximate size of common objects, such as the height of a door being two metres. Units are often not well understood or recognised; practical work can help pupils to gain familiarity with centimetres and metres.

Practical activities using a variety of different weighing scales and everyday objects are necessary to improve pupils' understanding of weight/mass.

Question 19: Levi's homework

In this question, pupils need to solve four missing number problems: an addition problem; a subtraction problem; a multiplication problem; and a division problem.

Pupils should be taught to solve addition, subtraction, multiplication and division missing number problems.

Pupils need practice in solving questions like this with the missing number in all possible positions, and they need to be encouraged to check that their answers work. They will also benefit from the opportunity to make up such problems for their fellow pupils to solve.

Question 20: Sharing

In this problem, pupils are asked to calculate the total number of sweets if each of nine children has four sweets and there is one left over.

If pupils are to use a more sophisticated method using multiplication, they need to know their tables and how and when to apply these number facts. Exploring different ways of solving such problems in class, with lots of discussion between pupils as they explain and evaluate the methods they use, is extremely valuable.

Questions 21 and 22: Authors

This question provides a pictogram showing pupils' favourite authors. The key shows that one icon represents 10 pupils. Pupils are asked to find the most popular author (question 21); how many chose Michael Rosen as their favourite (question 22a); how many took part in the survey (question 22b).

In the classroom, pupils enjoy, and benefit from, collecting their own data and representing it as pictograms, choosing appropriate keys for the information collected. They can also look through newspapers and magazines and on the internet for examples of pictograms to be discussed and interpreted in class.

Feedback to parents and carers

A report on the individual pupil is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the pupil at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual pupil in order to understand and maximise their potential.
- Explain that testing with *PTM*8 is part of the school's regular assessment regime and that all pupils in the year group(s) have been tested.
- As part of the test, pupils were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual pupil (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM*8.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our pupils to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 8, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.