Mathematics process categories

All of the UK curricula define multiple categories of mathematical proficiency that require pupils to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths' (PTM)* categories are based on the *Curriculum Aims* in the KS1, KS2 and KS3 National Curriculum for England (2013), and are also comparable with the GCSE Assessment Objectives: they adopt some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Pupils can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Pupils can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (for example, calculate with or compare decimal numbers, identify odd numbers, prime numbers and multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Pupils can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Pupils can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*, especially at younger ages where reading and English comprehension restrict the sorts of questions that can be asked. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

These tables show how the questions in *PTM*7 map onto these process categories.

| Paper test | | | |
|---|---|--|--|
| Process category | | | |
| FF: Fluency in facts and procedures | 1a, 2a, 3a, 4a, 4b | | |
| FC: Fluency in conceptual understanding | 1b, 1c, 2b, 2d, 3b, 6a, 8a, 9a, 9b | | |
| MR: Mathematical reasoning | 2c, 2e, 3c, 3e, 4c, 5a, 5b, 6b, 7a, 7b, 7c, 7d, 8b, 8c, 10a, 10b | | |
| PS: Problem solving | 1d, 3d, 5c, 10c | | |

| Digital test | | | |
|---|--|--|--|
| Process category | | | |
| FF: Fluency in facts and procedures | 1, 2, 6, 11, 16, 17, 18 | | |
| FC: Fluency in conceptual understanding | 3, 4, 7, 9, 12, 24, 25, 26, 34, 35, 38, 39, 40 | | |
| MR: Mathematical reasoning | 8, 10, 13, 15, 19, 20, 21, 22, 27, 28, 29, 30, 31, 32, 33, 36, 37, 41, 42 | | |
| PS: Problem solving | 5, 14, 23 | | |

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories shown above are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

| Wales | Closest PTM process categories | | | |
|--------------------------------|--------------------------------|----|----|----|
| Foundation Phase Skills | FF | FC | MR | PS |
| 1. Solve mathematical problems | | | | • |
| 2. Communicate mathematically | | • | • | |
| 3. Reason mathematically | | • | • | |
| Foundation Phase Range | • | | | |

| Northern Ireland | Closest PTM process categories | | | |
|--------------------------------------|--------------------------------|----|----|----|
| Key Stage 1 Processes in Mathematics | FF | FC | MR | PS |
| Making and monitoring decisions | | | | • |
| Communicating mathematically | | • | ٠ | |
| Mathematical reasoning | | • | ٠ | • |
| Individual mathematical topics | • | | | |

| Scotland * | Closest PTM process categories | | | |
|---|--------------------------------|----|----|----|
| Experiences and outcomes | FF | FC | MR | PS |
| develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work | | | ٠ | ٠ |
| engage with more abstract mathematical concepts and develop important new kinds of thinking | | | • | |
| understand the application of mathematics, its impact on our society past and present, and its potential for the future | | | | |
| develop essential numeracy skills which will allow me to participate fully in society | • | | | |
| establish firm foundations for further specialist learning | • | • | | |
| understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills | | | • | • |
| interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions | | | | • |
| apply skills and understanding creatively and logically to solve problems, within a variety of contexts | | | • | • |
| appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts | | | | |

^{*} Education Scotland 'Curriculum for Excellence: Numeracy and Mathematics' 14 May 2009. Accessed: 31 July 2014. <u>www.curriculumforexcellencescotland.gov.uk</u>

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the pupils in the relevant age group. The activities may therefore be used to obtain diagnostic information about each pupil's strengths and weaknesses, and may also be used to provide a basis from which pupils' mathematical understanding may be further developed.

This section discusses some of the ways in which pupils may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to pupils and discussing the activities in which they did well, in addition to those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the Pupil Booklet and the Teacher's script in the At a Glance Guide when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the overall performance of your pupils, and a basis for progress monitoring. This section is intended to help you identify the specific difficulties that pupils have with individual questions, and to suggest possible activities to help guide your future teaching.

Paper Test

Question 1: The month of July

A page of a calendar for the month of July is shown with some numbers missing. Pupils are asked to fill in the missing numbers and write / select the date for the third Thursday (part a); the numbers of Mondays in the month (part b); the day of the week for the 16th of July (part c). For part d, two clock faces are shown and pupils are asked to draw/move hands on the second clock to show one-and-a-half hours after 3 o'clock.

It is useful to spend time looking at number patterns and sequences in everyday life as well as on a number line and in number squares. Also there are some excellent activities on the internet. It would be helpful to have a clock highly visible in the classroom so that constant references can be made to everyday time-telling for pupils who need practice in this.

Question 2: Paws

In this question pupils need to solve problems using addition and subtraction and remember multiplication and division facts for the 5 and 10 multiplication tables. Five calculations with missing numbers are shown. Pupils are asked to find the missing number in an addition sum (part a); find the answer to a two-digit addition calculation (part b); find the missing number in a subtraction calculation (part c); find the missing number in a multiplication by 5 calculation (part d); find the missing number which divided by 10 gives the answer 8 (part e).

Pupils should be encouraged on a regular basis to check their own calculations by using inverse procedures. Puzzle questions such as 'What number am I thinking of? If I take away 21 and the answer is 43. If I divide by 10 and the answer is 7.'

Question 3: Shoe sizes

The shoe sizes task assesses whether pupils can interpret a pictogram and answer simple questions by counting a number of objects, in this case, the number of shoes in each category. Pupils also need to know how to total and compare data.

A pictogram showing how many children take which shoe size, from size 1 to 4, is provided. Pupils are asked how many children take size 3 shoes (part a); how many children take the smallest shoe size (part b); how many more take size 2 than 4 (part c); how many children took part in the survey (part d); to write 'R' for Ruby in the size 2 column (part e).

Practice at creating their own pictograms and writing questions to pose to their peers can be an enjoyable and worthwhile activity.

Question 4: Colouring patterns

In part a, a diagram made from polygons is shown and pupils are asked to identify a hexagon, and a square. In part b, a different diagram is shown and pupils are asked to count the number of triangles. In part c, five polygons are shown; pupils are asked to find a hexagon with a line of symmetry and draw in its line of symmetry, then find two pentagons with lines of symmetry and draw in their lines of symmetry.

Some pupils do find symmetry a difficult concept, so it could be worth buying a geometry tool (search online for 'Maths geometry tool'). The plastic tool has the reflective quality of a mirror as well as a transparent quality. By placing the tool on any shape the pupils can see symmetry and congruence. Moreover, rotations, reflections, flips and slides are easy to see.

Question 5: Amazing animals

This question asks pupils to extract information from a table and answer three simple questions based on the data. A table showing the speeds of some of the fastest animals is provided. Pupils are asked to use the table to find how much faster an antelope can run than a gazelle (part a); how much slower is a hare than a cheetah (part b); which animal is 10 mph slower than an antelope (part c).

Pupils could practice gathering data, constructing their own tables and writing questions based on them to be posed to their peers. Topics such as: how they travel to school (walk, car, bus), favourite days of the week, number of letters in popular names, our pets, and so on would encourage pupils to work with data in meaningful contexts.

Question 6: Number game

The number game tasks assess pupils' fluency in the use of numbers and their conceptual understanding of 'even' and 'odd 'numbers, as well as their understanding of the terms 'more than' and 'less than' in a problem-solving situation.

In part a of this task, a 3×3 number grid is shown: pupils are asked to identify an even number more than 10, an odd number less than 5, and a number 10 more than 7. In part b, a different 3×3 number grid is shown with two empty squares. Pupils are asked to write a number 5 less than 20 in the middle square and a number 10 more than 80 in the corner square.

Card games can be fun for children. The games can involve odd and even numbers, problems which emphasise the value of each digit in two-digit numbers and questions which foster the understanding of zero as a place holder.

Question 7: The school shop

This task assesses pupils' ability to recognise and know the value of different denominations of coins. They also need to be able to find different combinations of coins that equal the same amounts of money.

Four shopping situations are presented. In part a, pupils are asked to choose two of five coins to buy an apple; in part b, they choose three from six coins to buy a banana; in part c, they choose four of six coins to buy a carton of milk and find how much more it costs to buy milk than a banana; in part d, they find the change from 50p after spending 25p.

Although nothing can replace practice in using real money in real situations, classroom shopping situations using plastic coins and empty packages of everyday goods can be extremely helpful for pupils. Also, there are lots of activities available on the internet.

Question 8: Quiz

The quiz task asks pupils to recognise the place value of each digit in a two-digit number and to compare and order numbers from 58 and 71. Pupils need to subtract with two two-digit numbers and use the signs < and >. In this task four teams have had a quiz and we are shown their scores. Pupils are asked to draw a line from the words 'lowest' and 'highest' to the lowest and highest numerical scores (part a); and find the difference between these two scores (part b); then write the symbols < and > correctly between two scores (part c).

Using the 'less than' and 'more than' signs can be confusing for some children and it is often useful to ask the class as a whole how they can remember which is which.

Question 9: Cats

This task asks pupils to recognise and find $\frac{1}{2}$ and $\frac{1}{4}$ of a set of cats and to double the number 3. In part a, eight unshaded kittens are shown: pupils are asked to shade in half of the kittens and draw rings around one quarter of them. In part b, three tins of cat food are shown and pupils are asked how many tins there are in double this amount.

Pupils should be able to recognise, find, name and write the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ of a length, shape, set of objects or quantity.

Question 10: Tropical fish

The final question in this test is a problem-solving task that is a little more challenging. It asks pupils to solve problems based on a pattern which can be solved using simple addition, multiplication and division. A diagram of a baby fish (one stripe) and a one-year-old fish (three stripes) is shown. In part a, pupils are shown a fish with five stripes and are asked how old it is; in part b, they are asked how many stripes a fish will have when it is three years old; in part c, a fish with 11 stripes is shown and pupils are asked how old it is.

One way to solve this problem requires pupils to count 11 stripes, subtract one and divide by two, to find the answer 5. Another way to find a correct solution is to count on in twos from previous parts of the task. A further way might be to draw pictures of the fish at different ages.

It is worthwhile introducing problems such as this, since they can be thoughtprovoking for all pupils and can provide insightful discussion points for the whole class, particularly when different ways of solving each problem are compared.

Digital Test

Questions 1, 2, 3, 4 and 5: The month of July

A page of a calendar for the month of July is shown with some numbers missing. Pupils are asked to fill in the missing numbers and select the date for the third Thursday; the numbers of Mondays in the month; the day of the week for the 16th of July. In question 5, a clock face is shown and pupils are asked to move hands on the clock to show one-and-a-half hours after 3 o'clock.

It is useful to spend time looking at number patterns and sequences in everyday life as well as on a number line and in number squares. Also there are some excellent activities on the internet. It would be helpful to have a clock highly visible in the classroom so that constant references can be made to everyday time-telling for pupils who need practice in this.

Questions 6, 7, 8, 9 and 10: Paws

In this question pupils need to solve problems using addition and subtraction and remember multiplication and division facts for the 5 and 10 multiplication tables. Five calculations with missing numbers are shown. Pupils are asked to find the missing number in an addition sum (question 6); find the answer to a two-digit addition calculation (question 7); find the missing number in a subtraction calculation (question 8); find the missing number in a multiplication by 5 calculation (question 9); find the missing number which divided by 10 gives the answer 8 (question 10).

Pupils should be encouraged on a regular basis to check their own calculations by using inverse procedures. Puzzle questions such as 'What number am I thinking of? If I take away 21 and the answer is 43. If I divide by 10 and the answer is 7.'

Questions 11, 12, 13, 14 and 15: Shoe sizes

The shoe sizes task assesses whether pupils can interpret a pictogram and answer simple questions by counting a number of objects, in this case, the number of shoes in each category. Pupils also need to know how to total and compare data. A pictogram showing how many children take which shoe size, from size 1 to 4, is provided. Pupils are asked how many children take size 3 shoes (question 11); how many children take the smallest shoe size (question 12); how many more take size 2 than 4 (question 13); how many children took part in the survey (question 14); to identify the correct column for Ruby's shoe size (question 15). Practice at creating their own pictograms and writing questions to pose to their peers can be an enjoyable and worthwhile activity.

Questions 16, 17, 18, 19 and 20: Colouring patterns

In questions 16 and 17, a diagram made from polygons is shown and pupils are asked to identify a hexagon, and a square. In question 18, a different diagram is shown and pupils are asked to count the number of triangles. In question 19, five polygons are shown; pupils are asked to find a hexagon with a line of symmetry

then move the line to the hexagon to show where the line of symmetry is. In question 20; pupils are asked to find two pentagons with lines of symmetry and move the lines to the pentagons to show the lines of symmetry.

Some pupils do find symmetry a difficult concept, so it could be worth buying a geometry tool (search online for 'Maths geometry tool'). The plastic tool has the reflective quality of a mirror as well as a transparent quality. By placing the tool on any shape the pupils can see symmetry and congruence. Moreover, rotations, reflections, flips and slides are easy to see.

Questions 21, 22 and 23: Amazing animals

This question asks pupils to extract information from a table and answer three simple questions based on the data. A table showing the speeds of some of the fastest animals is provided. Pupils are asked to use the table to find how much faster an antelope can run than a gazelle (21); how much slower is a hare than a cheetah (22); which animal is 10 mph slower than an antelope (23).

Pupils could practice gathering data, constructing their own tables and writing questions based on them to be posed to their peers. Topics such as: how they travel to school (walk, car, bus), favourite days of the week, number of letters in popular names, our pets, and so on would encourage pupils to work with data in meaningful contexts.

Questions 24, 25, 26, 27 and 28: Number game

The number game tasks assess pupils' fluency in the use of numbers and their conceptual understanding of 'even' and 'odd 'numbers, as well as their understanding of the terms 'more than' and 'less than' in a problem-solving situation.

In questions 24, 25 and 26 of this task, a 3×3 number grid is shown: pupils are asked to identify an even number more than 10, an odd number less than 5, and a number 10 more than 7. In questions 28 and 29, a different 3×3 number grid is shown with two empty squares. Pupils are asked to write a number 5 less than 20 in the middle square and a number 10 more than 80 in the corner square.

Card games can be fun for children. The games can involve odd and even numbers, problems which emphasise the value of each digit in two-digit numbers and questions which foster the understanding of zero as a place holder.

Questions 29, 30, 31, 32 and 33: The school shop

This task assesses pupils' ability to recognise and know the value of different denominations of coins. They also need to be able to find different combinations of coins that equal the same amounts of money.

^{*} If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.

Four shopping situations are presented. In question 29, pupils are asked to choose two of five coins to buy an apple; in question 30, they choose three from six coins to buy a banana; in questions 31 and 32, they choose four of six coins to buy a carton of milk and find how much more it costs to buy milk than a banana; in question 33, they find the change from 50p after spending 25p.

Although nothing can replace practice in using real money in real situations, classroom shopping situations using plastic coins and empty packages of everyday goods can be extremely helpful for pupils. Also, there are lots of activities available on the internet.

Questions 34, 35, 36 and 37: Quiz

In this task four teams have had a quiz and we are shown their scores. Pupils are asked to identify the teams with the highest and lowest scores (questions 34 and 35); and find the difference between these two scores (question 36). In question 37 pupils are shown two symbols and told that the top symbol (>) means more than and the bottom symbol (<) means less than; they are then asked to move the correct teams into the empty boxes.

Using the 'less than' and 'more than' signs can be confusing for some children and it is often useful to ask the class as a whole how they can remember which is which.

Questions 38, 39 and 40: Cats

This task asks pupils to recognise and find 1/2 and 1/4 of a set of cats and to double the number 3. In questions 38 and 39, eight kittens are shown: pupils are asked to identify half of the kittens and then one quarter of them. In question 40, three tins of cat food are shown and pupils are asked how many tins there are in double this amount.

Pupils should be able to recognise, find, name and write the fractions 1/3, 1/4, 2/4, and 3/4 of a length, shape, set of objects or quantity.

Questions 41 and 42: Tropical fish

The final questions in this test involve a problem-solving task that is a little more challenging. They ask pupils to solve problems based on a pattern which can be solved using simple addition, multiplication and division. In question 41 a diagram of a baby fish (one stripe) and a one-year-old fish (three stripes) is shown, pupils are then shown a diagram of a fish with 5 stripes and are asked how old it is. In question 42 pupils are asked how many stripes a three-year old fish will have.

One way to find a correct solution is to count on in twos from previous parts of the task. A further way might be to draw pictures of the fish at different ages.

It is worthwhile introducing problems such as this, since they can be thoughtprovoking for all pupils and can provide insightful discussion points for the whole class, particularly when different ways of solving each problem are compared.

Feedback to parents and carers

A report on the individual pupil is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the pupil at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual pupil in order to understand and maximise their potential.
- Explain that testing with *PTM*7 is part of the school's regular assessment regime and that all pupils in the year group(s) have been tested.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual pupil (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM*7.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our pupils to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 7, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included^{*}. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]