

Mathematics process categories

PT MATHS 10

All of the UK curricula define multiple categories of mathematical proficiency that require pupils to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths' (PTM)* categories are based on the *Curriculum Aims* in the KS1, KS2 and KS3 National Curriculum for England (2013), and are also comparable with the GCSE Assessment Objectives: they adopt some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Pupils can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Pupils can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (for example, calculate with or compare decimal numbers, identify odd numbers, prime numbers and multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Pupils can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Pupils can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*, especially at younger ages where reading and English comprehension restrict the sorts of questions that can be asked. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

This table shows how the questions in *PTM10* map onto these process categories.

Process category	Mental Maths	Applying and Understanding Maths
FF: Fluency in facts and procedures	1, 2, 10, 11, 15	5a, 16a, 19a, 19b, 19c
FC: Fluency in conceptual understanding	3, 5, 6, 8, 9, 12,14	1, 2a, 3b, 4, 7a, 8, 9a, 12a, 15a, 16b, 17a, 17c, 18a, 18b, 19d, 20a
MR: Mathematical reasoning	4, 7, 13	2b, 5b, 6a, 6b, 7b, 9b, 11, 12b, 14, 15b, 16c, 17b, 18c, 20b
PS: Problem solving		2c, 3a, 6c, 10, 13

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories shown above are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest <i>PTM</i> process categories			
Key Stage 2 Skills	FF	FC	MR	PS
1. Solve mathematical problems				•
2. Communicate mathematically		•	•	
3. Reason mathematically		•	•	
Foundation Phase Range	•			

Northern Ireland	Closest <i>PTM</i> process categories			
Key Stage 2 Processes in Mathematics	FF	FC	MR	PS
Making and monitoring decisions				•
Communicating mathematically		•	•	
Mathematical reasoning		•	•	•
Individual mathematical topics	•			

Scotland *	Closest PTM process categories			
Experiences and outcomes	FF	FC	MR	PS
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

* Education Scotland 'Curriculum for Excellence: Numeracy and Mathematics' 14 May 2009.
 Accessed: 31 July 2014. www.curriculumforexcellencescotland.gov.uk

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the pupils in the relevant age group. The activities may therefore be used to obtain diagnostic information about each pupil's strengths and weaknesses, and may also be used to provide a basis from which pupils' mathematical understanding may be further developed.

This section discusses some of the ways in which pupils may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to pupils and discussing the activities in which they did well, in addition to those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the Pupil Booklet and the Teacher's script in the At a Glance Guide when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the overall performance of your pupils, and a basis for progress monitoring. This section is intended to help you identify the specific difficulties that pupils have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test pupils' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills and are relying too heavily on written methods for even simple sums. They may lack the confidence to recall mathematical facts under time pressure.

Regular quick-fire quizzes may help pupils gain fluency and confidence, and there are many software products that allow students to practise skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help pupils gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Applying and Understanding Maths test

Question 1: Running

Provided with a chart showing the distance David runs each day, pupils are asked to find the total distance he ran on two days (part a); how much further he ran on Tuesday than Wednesday (part b); then to change the distance he ran on Sunday from kilometres to metres (part c).

Pupils should be provided with an opportunity to work with decimals in a variety of ways, using practical activities and structured apparatus, to gain understanding and confidence. Doing decimal calculations using an empty number line may help some pupils.

Part c combines two difficult aspects of mathematics: decimals and the use of units. Pupils need to be secure in multiplying and dividing by powers of ten to be able to convert from one metric unit to another successfully.

Pupils need to understand the relationships between the units, which can only be achieved by working with the units in a variety of ways over a period of time until the relationships become familiar. Pupils can measure their own heights (and other lengths) and express them in centimetres and in metres (and maybe millimetres and kilometres for fun!), similarly, for weights in grams and kilograms; volumes in millilitres and litres; and money in pence and pounds. Pupils should explore what the prefixes kilo-, centi-, milli-, and so on, mean and look for examples. Pupils also need to understand approximate relationships between metric and imperial units.

Question 2: Cats

In this task, pupils are provided with a table showing the names of four cats, together with their ages and weights. Pupils are asked to name the youngest cat (part a); find the difference in mass between the lightest and heaviest cats (part b); then to write the names of three of the cats on a graph showing mass and age (part c).

As mentioned previously, counting in decimals and using an empty number line may be useful. Pupils might also use scales to weigh and measure objects (maybe themselves!) and find differences between heights and masses.

Question 3: Books

In this task, pupils are told that Miss Begum has twenty-seven children in her class. She needs to buy a book for each of them and one for herself, but the books are sold in packs of five and each pack costs £12.10. Pupils are asked to find how many packs are needed (part a), then how much they cost (part b).

In this question, pupils need to assimilate several simple items of information and work out that, to obtain twenty-eight books, Miss Begum needs to buy six packs of five books (part a), then work out the total cost at £12.10 per pack (part b). At each stage in this task, pupils need to be able to concentrate on using the relevant pieces of information before moving on to the next part of the task.

In addition to being able to recall their multiplication tables, pupils should be provided with opportunities to discuss real-life problems in pairs or small groups, discussing how the answers can be calculated, then trying to convince one another.

Question 4: Jeans

Anne buys some jeans that are 0.91 metres long, but she wants them to be 0.85 metres long.

This question again involves subtraction of two decimals.

Question 5: Pencils, paper and pens

In this task, we are told that Mrs Harris gives each of her pupils one pencil, two pieces of paper, and three pens. We are asked to find how many of each she needs for ten pupils (part a), and then to identify which of the three graphs drawn shows the relationship between the number of children and the number of pencils, pieces of paper, and pens (part b).

In the classroom, pupils can discuss the interpretation of similar graphs – could they have identified the relevant graphs if the axes had been scaled, for example? There are many opportunities for mathematical reasoning and explanation here.

Question 6: Fruit and vegetables

In this task, pupils are provided with information about the weights of five different items of fruit. They are asked to find the minimum weight of four kiwi fruit (part a), the largest weight of seven items of fruit (part b), and the largest number of grapes in 250 grams (part c).

This question assesses pupils' ability to apply simple examples of upper and lower bounds in the semi-realistic context of the weight of different items of fruit.

Practice of actually weighing may help, but to succeed with this type of question pupils need to think rather abstractly (for example, 'What if each of the kiwi fruit weighed the least possible?'), so discussion and reasoning are essential.

Question 7: Winter

In this task, pupils are provided with a table showing the average winter temperatures of five different cities in the USA that are all below zero degrees Celsius. Pupils are asked to identify the warmest temperature (part a), then to find the temperature of London which is nine degrees higher than Helsinki (part b).

Pupils need to be able to interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero.

Classroom work with thermometers, heights above and below sea level, and number lines, particularly if pupils can move up and down a number line themselves to interpret the temperature differences, can provide valuable experience.

Question 8: Cuboid

In this task, pupils are provided with a diagram of a cuboid showing that its dimensions are 6 cm, 2 cm and 4 cm. The problem is to find how many centimetre cubes are used to make the cuboid and show their work.

Pupils can be set challenges such as finding how many different cuboids can be made using thirty-six cubes, which will help them to gain good spatial understanding, work systematically and develop numerical fluency.

Question 9: Equivalent fractions

In this question, five fractions with different denominators are provided. Pupils are asked to find one fraction that is not equivalent to the other four (part a), and write another fraction that is equivalent to the other four (part b).

Fractions are a common problem area for pupils. In spite of this, pupils should be able to recognise families of common equivalent fractions.

Practical work with fractions represented in different ways using fraction walls provides a good way of comparing fractions, seeing how many parts make a whole, and identifying equivalent fractions. Pupils can make fraction walls of their own using squared paper, or strips of paper folded to show the relevant fractions. (Why are some fractions easier to fold than others?) Equivalent fractions can also be identified on a number line. Pupils can list and generate sets of equivalent fractions and discuss the number relationships involved.

Question 10: Holiday camp

In this problem, pupils are told that there is a group of between 60 and 100 children: when put into pairs, one child is left, when put into groups of five one child is left, and when put into groups of eight there is one child left.

Pupils might begin by listing the numbers that satisfy each constraint. If they begin with the multiples of 8 plus one they get: 65, 73, 81, 89, 97. Only 81 is one more than a multiple of 5 and this is an odd number (one more than a pair). So all the constraints are satisfied.

This question and others like it can provide the basis of a good class activity; pupils might use a 10 by 10 number grid to identify possible numbers according to the given conditions and, finally, find the solution. In doing so, they can be encouraged to recognise and explain the patterns that the grid shows. For example, pupils might find all the numbers that are one more than two, find numbers that are one more than a multiple of five and find numbers that are one more than eight, before finding the solution to the problem.

Question 11: Rods

In this question, pupils are shown a simple representation of some rods on a square grid. They are asked which rod is 50 per cent of Rod A (part a); which rod is two thirds of Rod B (part b); and what fraction of Rod C is Rod A (part c).

This task assesses pupils' understanding of the concept of percentages as well as fractions.

Working with fractions of quantities can be done in many different ways, using counters, squared paper, rods, multilink, cakes, pizzas, and so on.

Question 12: Halves

In this task, pupils are provided with eight different fractions and asked to decide whether each fraction is less than, equal to, or more than a half (part a), then asked to make up one more fraction in each category (part b).

Classroom activities of the kind suggested in the previous question are useful in improving pupils' skills when working with fractions.

Question 13: Class trip

In this task, when Mr Shah's class goes on a trip, one adult is needed for every six pupils and there are twenty-eight pupils in the class. The problem is to find how many adults are needed on the trip. This task involves dividing twenty-eight by six and rounding up to interpret the result appropriately.

This question can lead to some useful class discussion as pupils consider different ways in which one number divided by another should be interpreted, depending on the context.

Suppose the problem is that four people spend £18 and want to share out the cost equally between them; that we have 18 sausages to share between four people; that we have 18 balloons to share between four children, and so on.

Question 14: T-tiles

In this task, pupils are shown a T-tile shape on a grid and three different shapes made from T-tiles. The problem is to find how the three shapes can be made using T-tiles.

There are many enjoyable and productive classroom activities where pupils can work with shapes (which they can design themselves) and patterns of shapes made from them. This can link with area and perimeter work; all the shapes in this question have the same area but their perimeters are 16, 14 and 14. Can pupils find shapes made from two T-tiles that have bigger or smaller perimeters?

Question 15: Party time

In this task thirty-six pupils are planning a party and they will all sit around tables: there will be the same number (more than four) at each table. The problem is to find all the ways of arranging people around tables (part a) then how many packs of crisps are needed, if there are eight bags in each pack (part b).

In part a, pupils need to be fluent enough with their multiplication tables to correctly find all four different ways to multiply two numbers together and get the answer thirty-six. In part b of the question, pupils need to divide thirty-six by eight and round up their answer. Working with a 12 by 12 multiplication grid can be a useful way to find answers to questions such as this set in real world situations.

Question 16: Staircases

This question uses staircases to stimulate work with number sequences. In part a of this 'staircases' question, the number sequence increases in jumps of five; in part b, the jumps are halves; in part c the jumps are four, but the starting point is three which is not in the four times table.

Number sequences are an aspect of mathematics that many pupils enjoy working on; pupils need to practice not just extending sequences, but also going backwards and filling in different numbers of missing terms. Sequences can be made using counters, squares on squared paper, and other simple apparatus. Working with sequences in a variety of different ways with fractions and decimal numbers can lead to richer understanding.

Question 17: Vegetable garden

In this task, pupils are provided with a diagram showing Fran's garden on a square grid; the dimensions of some of the sides are shown and others can be calculated. The problems are to find the perimeter of the garden (part a); draw a diagram of a different garden with the same area but a smaller perimeter (part b); then find the perimeter of the new garden (part c).

This question, in a quasi-realistic context, assesses pupils' understanding of area and perimeter.

In part a, the limited understanding of some pupils that finding perimeters means 'adding up all the numbers' may have meant that some lengths were missed. Practical work with squared paper can help pupils to move beyond a limited understanding of perimeters and find more reliable methods of calculating perimeters. Pupils can be encouraged to develop the understanding that the perimeter of a shape such as this is the same as that of the surrounding rectangle and to explain why.

Question 18: Cities

Pupils are provided with a table showing six cities and their distances from London. They are asked to find the city that is furthest from London (part a); to list the cities in order of distance from London (part b); then find how much further away from London is Honolulu than Los Angeles (part c).

This is a relatively straightforward question but it involves large numbers, the distances of some world cities from London.

Large numbers such as these occur frequently when hearing and reading about real-life situations. In the classroom, it is helpful to read about and discuss everyday situations involving large numbers. When doing calculations with large numbers, pupils should be encouraged to estimate their answers so that they can check that their calculated answer is sensible.

Question 19: Angles

Pupils are shown diagrams of five angles. The task is to identify the smallest angle (part a); the largest angle (part b); the reflex angles (part c); and then the two angles that together make a complete turn (part d).

Understanding an angle as a measure of a turn, rather than as a static measurement, is helpful.

All pupils can envisage turning a tap on and off. Many class activities, such as pupils standing up and rotating to face different directions, identifying whether the angle turned is acute, obtuse or reflex, and what further turn they need to make to get back to the original direction, can improve pupils' understanding about angles. Programming a robot to make turns can also be a valuable activity.

Question 20: Sunshine

This task provides a table showing the number of hours of sunshine each month. Pupils are asked to identify the month with the most hours of sunshine (part a); then to complete a horizontal bar chart showing the information presented in the table (part b).

Classroom activities or tasks that require pupils to explain the shape of graphs and compare similar graphs with different data can be useful in aiding their understanding.

Feedback to parents and carers

A report on the individual pupil is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the pupil at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information – either orally or in writing – explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual pupil in order to understand and maximise their potential.
- Explain that testing with *PTM10* is part of the school's regular assessment regime and that all pupils in the year group(s) have been tested.
- As part of the test, pupils were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual pupil (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM10*.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our pupils to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 10, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.